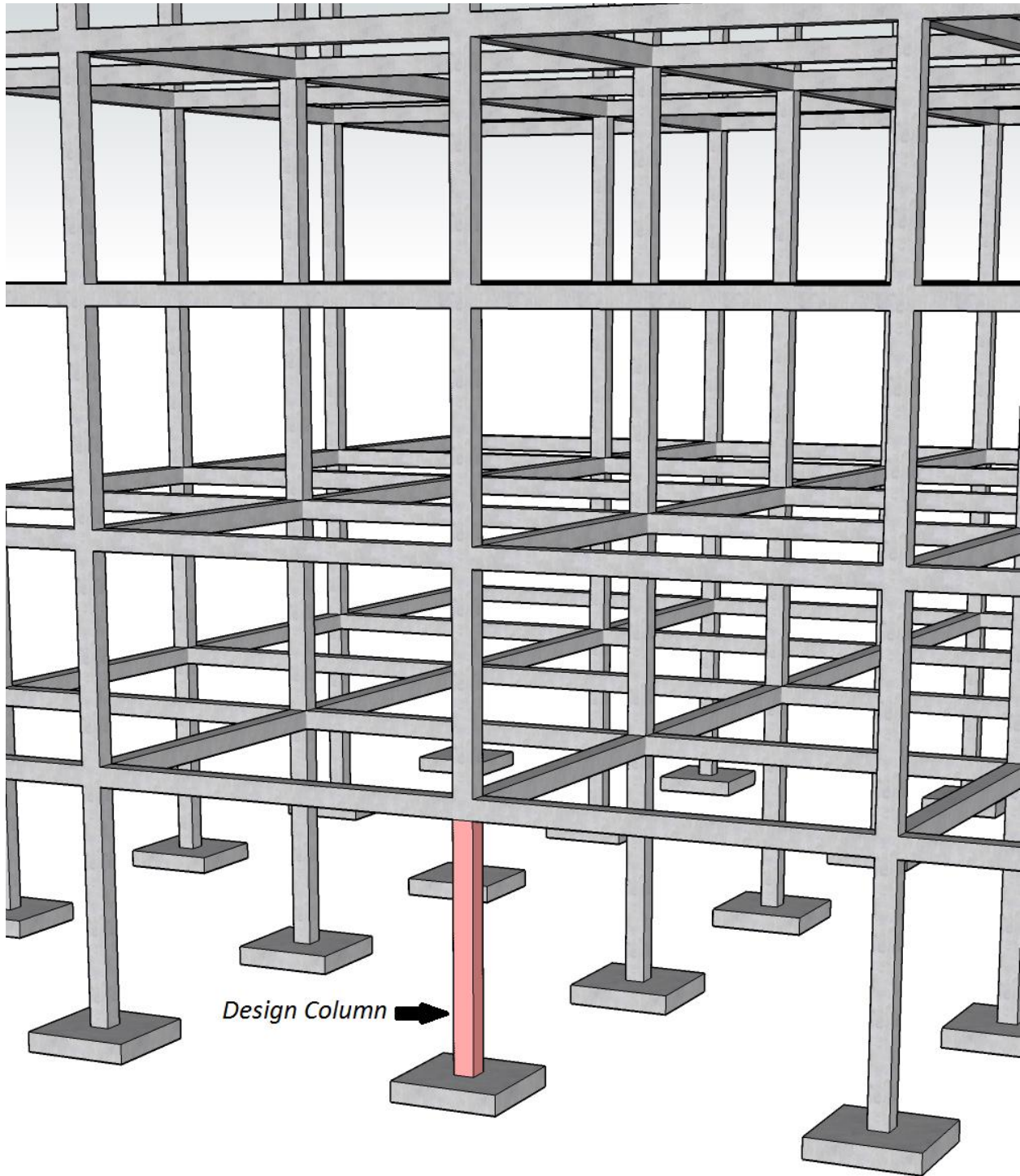


Slenderness Effects for Columns in Non-Sway Frame - Moment Magnification Method (ACI 318-19)



Slender Concrete Column Design in Non-Sway Frame Buildings

Evaluate slenderness effects for columns in a non-sway multistory reinforced concrete frame by determining the adequacy of the square tied column shown below, which is an exterior first floor column. The design forces obtained from a first-order analysis are provided in the design data section below. The story height is 12 ft. it is assumed that the frame is braced sufficiently to prevent relative translation of its joints. Assume 40% of the factored axial load is sustained. Compare the calculated results with exact values from [spColumn](#) engineering software program from [StructurePoint](#).

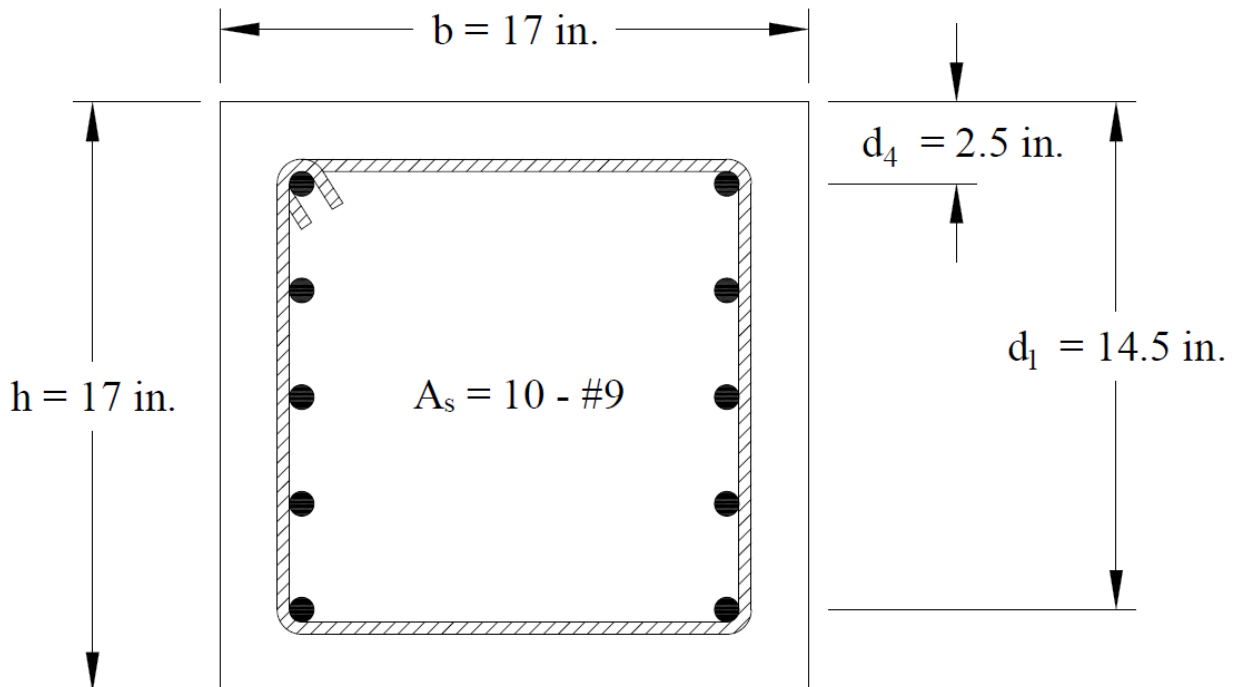


Figure 1 – Reinforced Concrete Column Cross-Section

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Code

Building Code Requirements for Structural Concrete (ACI 318-19) and Commentary (ACI 318R-19)

Reference

Reinforced Concrete Design, Eighth Edition, 2018, Wang C. et. al., Example 13.17.3.

Design Data

Concrete $f_c' = 3000$ psi

Steel $f_y = 60000$ psi

Beams: $h = 24$ in., $b = 14$ in., $l = 7$ m

Columns: $h = 17$ in., $b = 17$ in. $H = 12$ ft

factored design forces obtained from first-order analysis from the reference:

Table 1 - Column factored loads			
Load Case	Axial Load, kip	Bending Moment, kip.ft	
		Top	Bottom
Factored Load	1776*	105	0

* Assume 40% of the axial load is sustained

1. Slenderness Effects and Sway or Non-sway Frame Designation

Columns and stories in structures are considered as non-sway frames if the stability index for the story (Q) does not exceed 0.05. ACI 318-19 (6.6.4.3)

The reference assumed that the frame is a non-sway frame since Q value is less than 0.05.

2. Determine Slenderness Effects

The reference decided to be consistent with the more conservative procedure provided by **ACI 318-19 (6.6.4.4.3)** by taking k value equals to 1.0. However, the k value, in this example, is calculated based on the exact procedure for illustration purposes.

$$I_{column} = 0.7 \times \frac{c^4}{12} = 0.7 \times \frac{17^4}{12} = 4872 \text{ in.}^4 \quad \text{ACI 318-19 (6.6.3.1.1(a))}$$

$$I_{beam} = 0.35 \times \frac{b \times h^3}{12} = 0.35 \times \frac{14 \times 24^3}{12} = 5645 \text{ in.}^4 \quad \text{ACI 318-19 (6.6.3.1.1(a))}$$

$$E = 57,000 \times \sqrt{f'_c} = 57,000 \times \sqrt{3000} = 3122 \text{ ksi} \quad \text{ACI 318-19 (19.2.2.1.b)}$$

For columns:

$$\frac{E \times I_{column}}{l_c} = \frac{3122 \times 4872}{12 \times 12} = 8.8 \times 10^3 \text{ kip.ft}$$

For beams framing into the columns:

$$\frac{E \times I_{beam}}{l_b} = \frac{3122 \times 5645}{30 \times 12} = 4.08 \times 10^3 \text{ kip.ft}$$

$$\Psi_A = \frac{\left(\sum \frac{EI}{l_c} \right)_{columns}}{\left(\sum \frac{EI}{l} \right)_{beams}} = \frac{2 \times 8.80 \times 10^3}{4.08 \times 10^3} = 4.32 \quad \text{ACI 318-19 (Figure R6.2.5)}$$

$$\Psi_B = \infty \text{ (Column was assumed hinged at base)} \quad \text{ACI 318-19 (Figure R6.2.5)}$$

Using Figure R6.2.5 from ACI 318-19 $\rightarrow k = 0.959$ as shown in the figure below for the exterior columns with one beam framing into them in the directions of analysis.

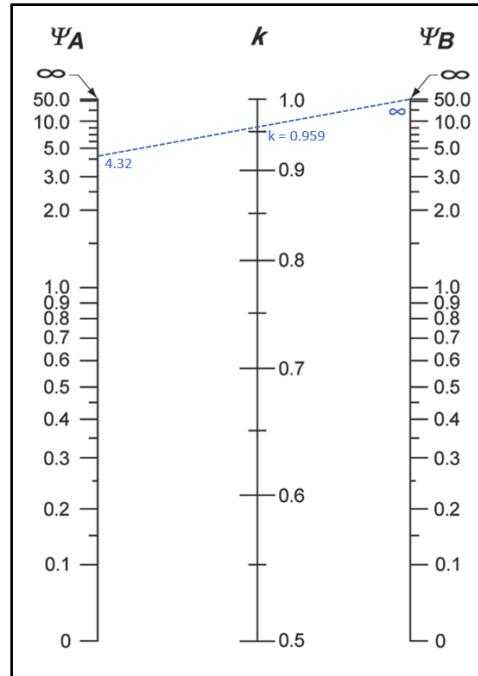


Figure 2 – Effective Length Factor (k) (Non-Sway Frame)

ACI 318-19 allows to neglect the slenderness in a non-sway frame if:

$$\frac{k \times l_u}{r} \leq 34 + 12 \left(\frac{M_1}{M_2} \right)$$

ACI 318-19 (6.2.5b)

Where:

$$r = \text{radius of gyration} = (a) \sqrt{\frac{I_g}{A_g}} \quad \text{or} \quad (b) 0.3 \times c_1$$

ACI 318-19 (6.2.5.1)

$$r = \sqrt{\frac{I_g}{A_g}} = \sqrt{\frac{c^2}{12}} = \sqrt{\frac{17^2}{12}} = 4.91 \text{ in.}$$

$$\frac{0.959 \times (12 \times 12 - 24)}{4.91} = 23.45 < 34 - 12 \left(\frac{0}{105} \right) = 34 \quad \therefore \text{slenderness can be neglected.}$$

Even though it is not required to consider slenderness effects for this column, the moment magnification method will be shown for illustration.

3. Moment Magnification – Non-Sway Frame

$$M_{c2} = \delta M_2 \quad \text{ACI 318-19 (6.6.4.5.1)}$$

Where:

$$\delta = \text{magnification factor} = \frac{C_m}{1 - \frac{P_u}{0.75P_c}} \geq 1.0 \quad \text{ACI 318-19 (6.6.4.5.2)}$$

$$P_c = \frac{\pi^2 (EI)_{eff}}{(kl_u)^2} \quad \text{ACI 318-19 (6.6.4.4.2)}$$

Where:

$$(EI)_{eff} = \left\{ \begin{array}{l} \text{(a) } \frac{0.4E_c I_g}{1 + \beta_{dns}} \\ \text{(b) } \frac{0.2E_c I_g + E_s I_{se}}{1 + \beta_{dns}} \\ \text{(c) } \frac{E_c I}{1 + \beta_{dns}} \end{array} \right\} \quad \text{ACI 318-19 (6.6.4.4.4)}$$

There are three options for calculating the effective flexural stiffness of slender concrete columns $(EI)_{eff}$. The second equation provides accurate representation of the reinforcement in the section and will be used in this example and is also used by the solver in [spColumn](#). Further comparison of the available options is provided in “[Effective Flexural Stiffness for Critical Buckling Load of Concrete Columns](#)” technical note.

3.1. Calculation of Critical Load (P_c)

$$I_{column} = \frac{c^4}{12} = \frac{17^4}{12} = 6960 \text{ in.}^4 \quad \text{ACI 318-19 (Table 6.6.3.1.1(a))}$$

$$E_c = 57,000 \times \sqrt{f'_c} = 57,000 \times \sqrt{3000} = 3122 \text{ ksi} \quad \text{ACI 318-19 (19.2.2.1.a)}$$

β_{dns} is the ratio of maximum factored sustained axial load to maximum factored axial load associated with the same load combination. ACI 318-19 (6.6.4.4.4)

In this example, it is assumed that 40% of the factored axial load is sustained.

$$\beta_{dns} = \frac{P_{u,sustained}}{P_u} = \frac{0.4 \times P_u}{P_u} = 0.40 < 1.00 \rightarrow \therefore \beta_{dns} = 0.40$$

With 10-#9 reinforcement equally distributed on two sides and 17 in. x 17 in. column section $\rightarrow I_{se} = 360 \text{ in.}^4$.

$$(EI)_{eff} = \frac{0.2E_c I_g + E_s I_{se}}{1 + \beta_{dns}} \quad \text{ACI 318-19 (6.6.4.4.4(b))}$$

$$(EI)_{eff} = \frac{0.2 \times 3122 \times 6960 + 29,000 \times 360}{1 + 0.4} = 10.56 \times 10^6 \text{ kip-in.}^2$$

$$P_c = \frac{\pi^2 \times 10.56 \times 10^6}{(0.959 \times (12 - 2) \times 12)^2} = 7871 \text{ kip}$$

3.2. Calculation of Magnified Moment (M_c)

$$C_m = 0.6 + 0.4 \frac{M_1}{M_2} \quad \text{ACI 318-19 (6.6.4.5.3a)}$$

$$C_m = 0.6 + 0.4 \left(\frac{0}{105} \right) = 0.6$$

$$\delta = \frac{C_m}{1 - \frac{P_u}{0.75 P_c}} \geq 1.0 \quad \text{ACI 318-19 (6.6.4.5.2)}$$

$$\delta = \frac{0.6}{1 - \frac{525}{0.75 \times 7871}} = 0.66 < 1.00 \rightarrow \delta = 1.00$$

$$M_{min} = P_u (0.6 + 0.03h) \quad \text{ACI 318-19 (6.6.4.5.4)}$$

Where $P_u = 525$ kip, and $h =$ the section dimension in the direction being considered = 17 in.

$$M_{min} = 525 \left(\frac{0.6 + 0.03 \times 17}{12} \right) = 48.56 \text{ kip.ft}$$

$$M_2 = 105 \text{ kip.ft} > M_{2,min} = 48.56 \text{ kip.ft} \rightarrow M_2 = 105 \text{ kip.ft} \quad \text{ACI 318-19 (6.6.4.5.4)}$$

$$M_{c2} = \delta M_2 \quad \text{ACI 318-19 (6.6.4.5.1)}$$

$$M_{c2} = 1.00 \times 105 = 105 \text{ kip.ft}$$

4. Column Design

Based on the factored axial loads and magnified moments considering slenderness effects, the capacity of the assumed column section (17 in. × 17 in. with 10 – #9 bars distributed on two sides) will be checked and confirmed to finalize the design. A column interaction diagram will be generated using strain compatibility analysis, the detailed procedure to develop column interaction diagram can be found in “[Interaction Diagram - Tied Reinforced Concrete Column](#)” example.

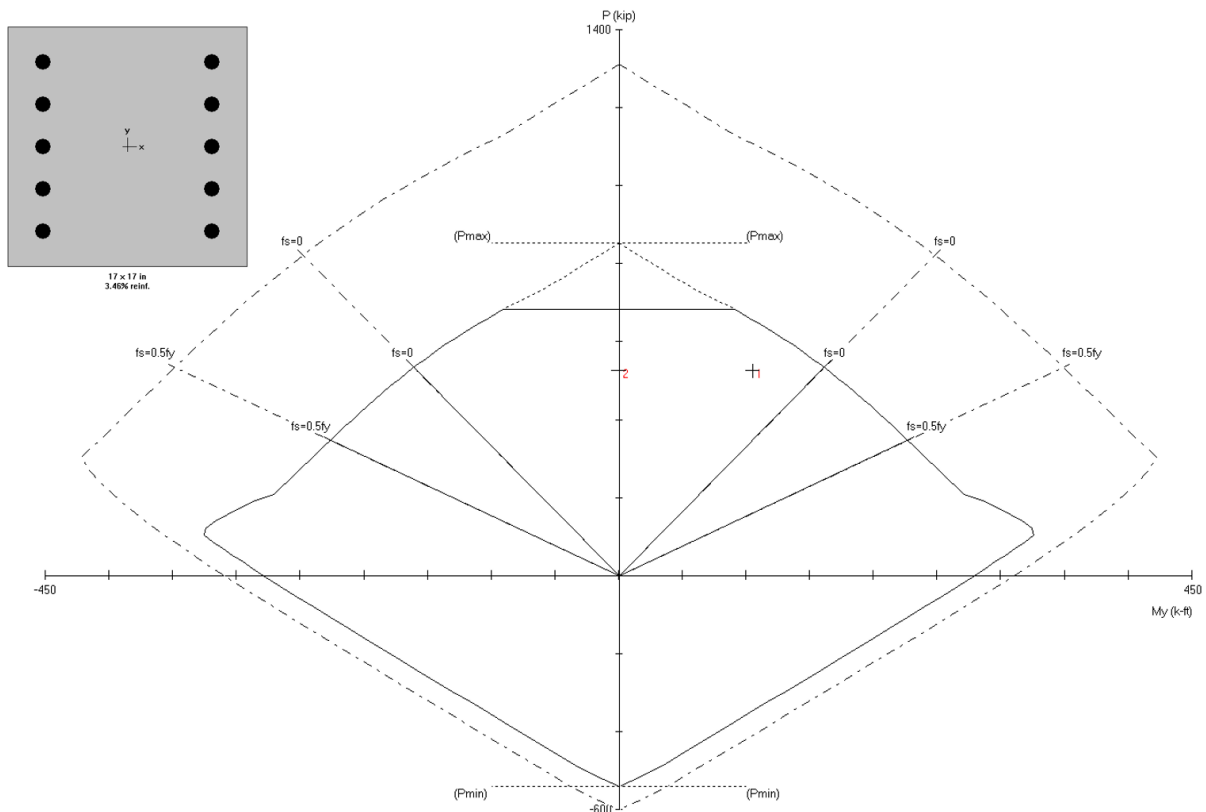


Figure 3 – Column Interaction Diagram

5. Column Design - spColumn Software

[spColumn](#) program performs the analysis of the reinforced concrete section conforming to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames. For this column section, we ran in design mode with control points using the ACI 318-19. The graphical and text results are provided below for both input and output of the [spColumn](#) model.

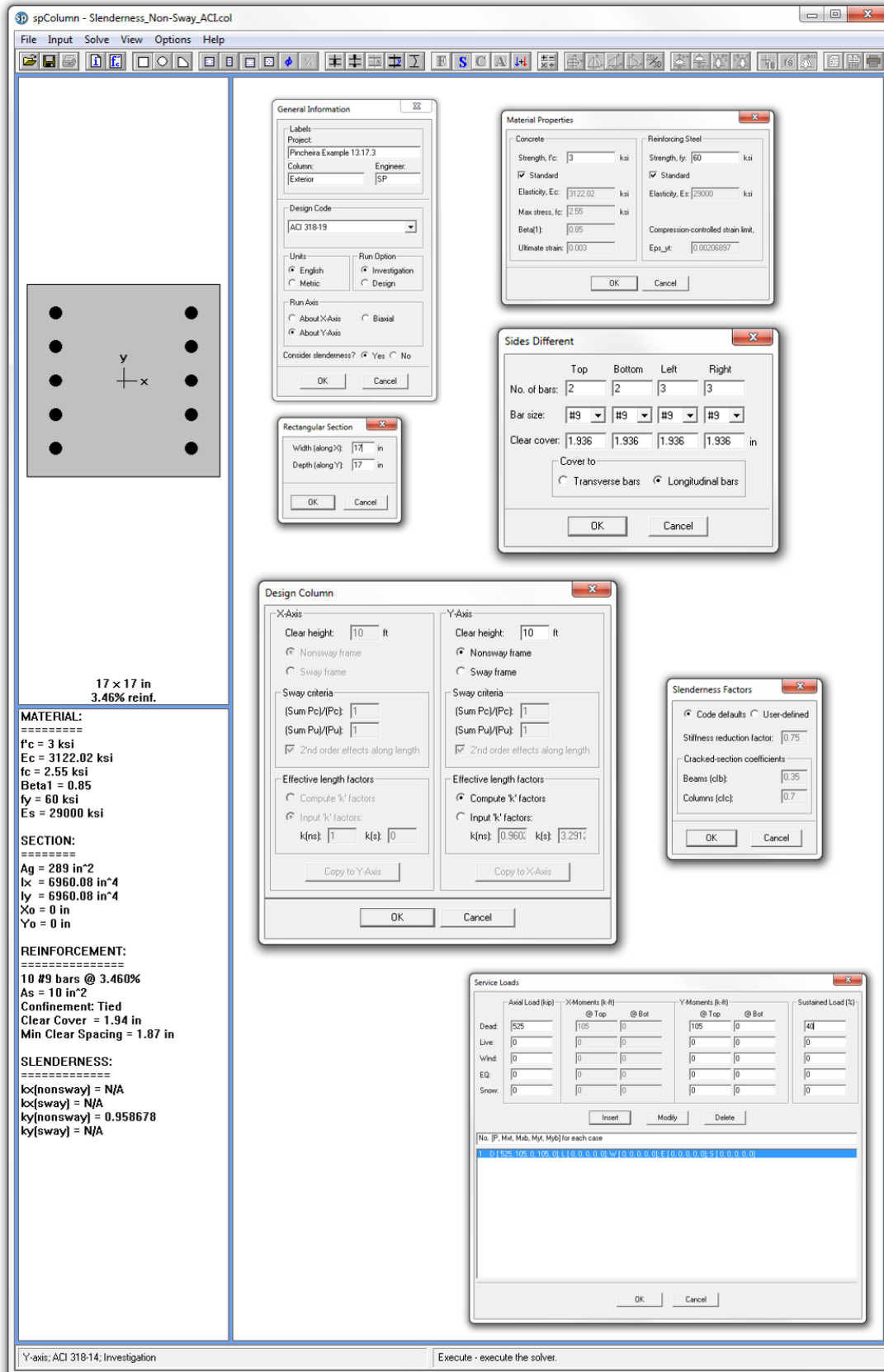


Figure 4 – spColumn Model Input Wizard Windows

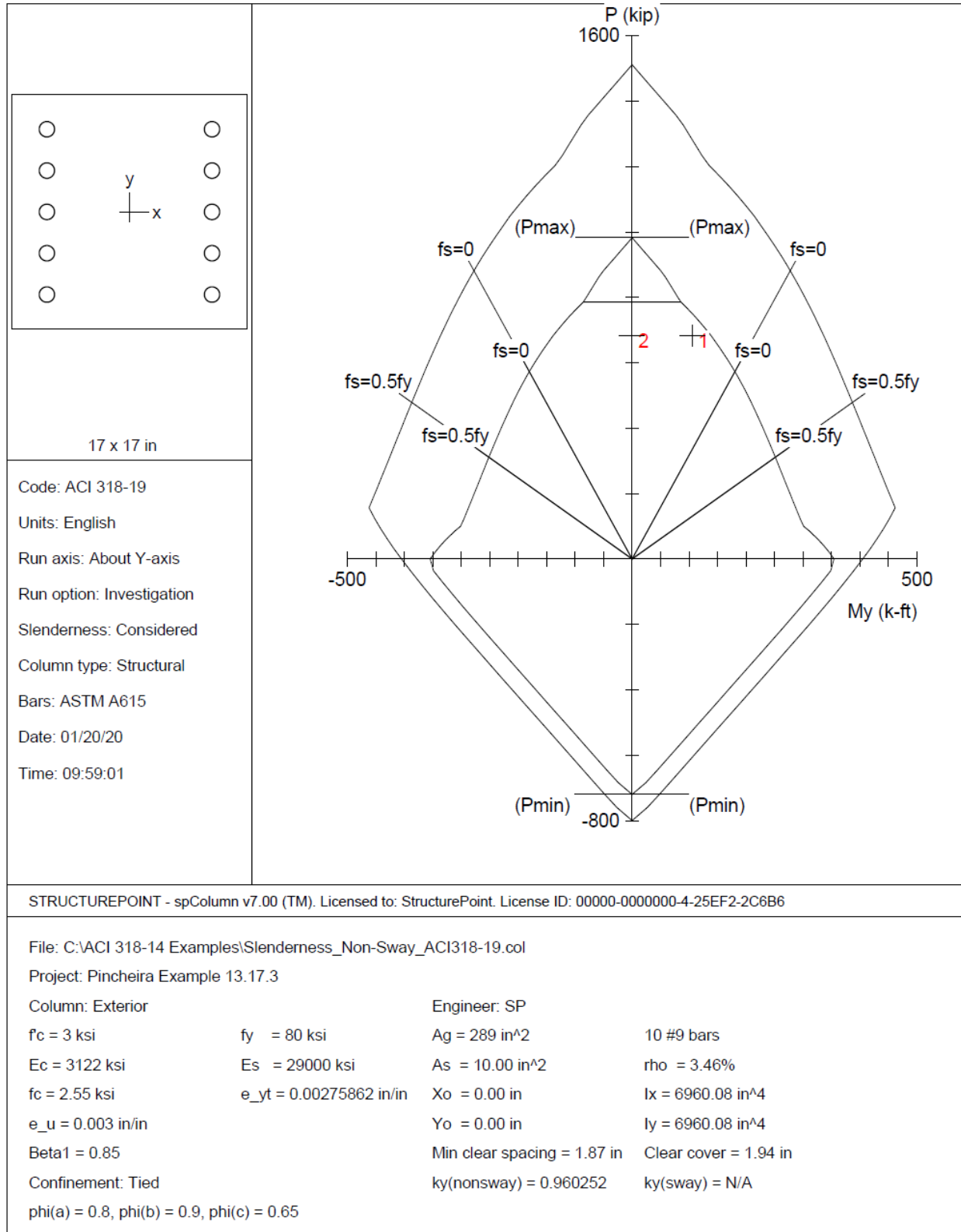
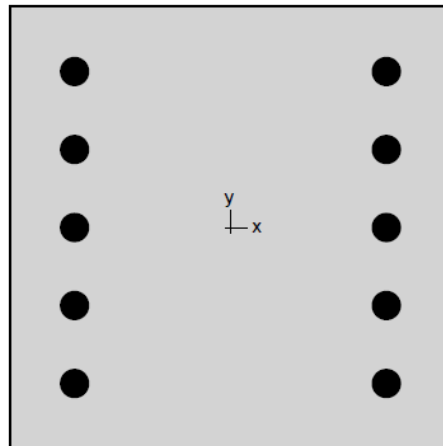


Figure 5 – Column Section Interaction Diagram about Y-Axis (spColumn)



spColumn v7.00
Computer program for the Strength Design of Reinforced Concrete Sections
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1. General Information

File Name	C:\ACI 318-14 Examples\Slenderness_Non-Sway_ACI318-19.col
Project	Pincheira Example 13.17.3
Column	Exterior
Engineer	SP
Code	ACI 318-19
Bar Set	ASTM A615
Units	English
Run Option	Investigation
Run Axis	Y - axis
Slenderness	Considered
Column Type	Structural
Capacity Method	Critical capacity

2. Material Properties

2.1. Concrete

Type	Standard
f_c	3 ksi
E_c	3122.02 ksi
f_o	2.55 ksi
ϵ_u	0.003 in/in
β_1	0.85

2.2. Steel

Type	Standard
f_y	80 ksi
E_s	29000 ksi
ϵ_{yt}	0.00275862 in/in

3. Section

3.1. Shape and Properties

Type	Rectangular
Width	17 in
Depth	17 in
A_g	289 in ²
I_x	6960.08 in ⁴
I_y	6960.08 in ⁴
r_x	4.90748 in
r_y	4.90748 in
X_o	0 in
Y_o	0 in

3.2. Section Figure

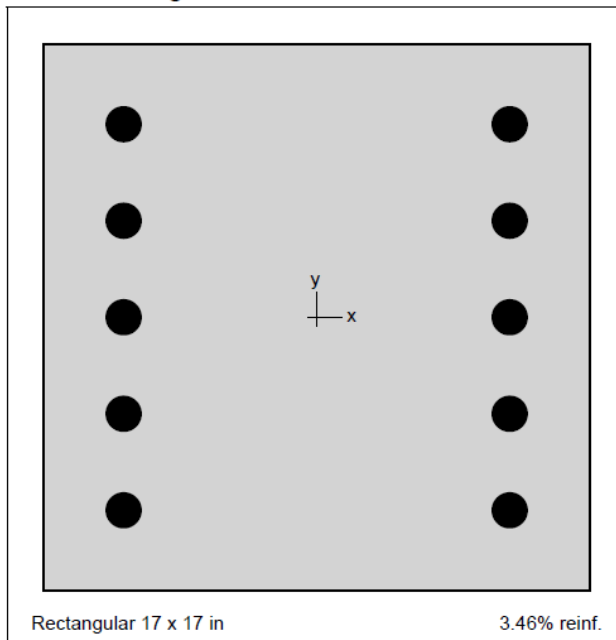


Figure 1: Column section

4. Reinforcement

4.1. Bar Set: ASTM A615

Bar	Diameter in	Area in ²	Bar	Diameter in	Area in ²	Bar	Diameter in	Area in ²
#3	0.38	0.11	#4	0.50	0.20	#5	0.63	0.31
#6	0.75	0.44	#7	0.88	0.60	#8	1.00	0.79
#9	1.13	1.00	#10	1.27	1.27	#11	1.41	1.56
#14	1.69	2.25	#18	2.26	4.00			

4.2. Confinement and Factors

Confinement type	Tied
For #10 bars or less	#3 ties
For larger bars	#4 ties
Capacity Reduction Factors	
Axial compression, (a)	0.8
Tension controlled ϕ , (b)	0.9
Compression controlled ϕ , (c)	0.65

4.3. Arrangement

Pattern	Sides different
Bar layout	Rectangular
Cover to	Longitudinal bars
Clear cover	---
Bars	---

Total steel area, A_s	10.00 in ²
Rho	3.46 %
Minimum clear spacing	1.87 in

4.4. Bars Provided

	Bars	Clear cover in
Top	2 #9	1.936
Bottom	2 #9	1.936
Left	3 #9	1.936
Right	3 #9	1.936

5. Loading

5.1. Load Combinations

Combination	Dead	Live	Wind	EQ	Snow
U1	1.000	0.000	0.000	0.000	0.000

5.2. Service Loads

No.	Load Case	Axial Load kip	Mx @ Top k-ft	Mx @ Bottom k-ft	My @ Top k-ft	My @ Bottom k-ft
1	Dead	682.50	105.00	0.00	105.00	0.00
1	Live	0.00	0.00	0.00	0.00	0.00
1	Wind	0.00	0.00	0.00	0.00	0.00
1	EQ	0.00	0.00	0.00	0.00	0.00
1	Snow	0.00	0.00	0.00	0.00	0.00

5.3. Sustained Load Factors

Load Case	Factor %
Dead	40
Live	0
Wind	0
EQ	0
Snow	0

6. Slenderness

6.1. Sway Criteria

Y-Axis	Non-sway column
--------	-----------------

6.2. Columns

Column	Axis	Height ft	Width in	Depth in	I in ⁴	f'_c ksi	E_c ksi	
Design	Y	10	17	17	6960.08	3	3122.02	
Above	Y	12	17	17	6960.08	3	3122.02	
Below	Y	(no column specified...)						

6.3. Y - Beams

Beam	Length ft	Width in	Depth in	I in ⁴	f'_c ksi	E_c ksi
Above Left	30	14	24	16128	3	3122.02

Beam	Length ft	Width in	Depth in	I in ⁴	f _c ksi	E _c ksi
Above Right	(no beam specified...)					
Below Left	(no beam specified...)					
Below Right	(no beam specified...)					

7. Moment Magnification

7.1. General Parameters

Factors	Code defaults
Stiffness reduction factor, ϕ_k	0.75
Cracked section coefficients, c1(beams)	0.35
Cracked section coefficients, c1(columns)	0.7
0.2 E _c I _g + E _s I _{se} (Y-axis)	1.48e+007 kip-in ²
Minimum eccentricity, e _{y min}	1.11 in

7.2. Effective Length Factors

Axis	Ψ_{top}	Ψ_{bottom}	k (Nonsway)	k (Sway)	kl _u /r
Y	4.512	999.000	0.960	(N/A)	23.48

7.3. Magnification Factors: Y - axis

* Slenderness need not be considered.

Load Combo	At Ends						Along Length					
	ΣP_u kip	P _c kip	ΣP_c kip	β_{ds}	δ_s	P _u kip	k'I _u /r	P _c kip	β_{dns}	C _m	δ	
1 U1	(N/A)	(N/A)	(N/A)	(N/A)	(N/A)	682.50	(N/A)	7850.31	0.400	(N/A)	(N/A) *	

8. Factored Moments

NOTE: Each loading combination includes the following cases:

Top - At column top
Bot - At column bottom

8.1. Y - axis

Load Combo	1 st Order					2 nd Order			Ratio 2 nd /1 st
	M _{ns} k-ft	M _s k-ft	M _u k-ft	M _{min} k-ft	M _i k-ft	M _c k-ft			
1 U1 Top	105.00	(N/A)	105.00	(N/A)	M ₂ =	(N/A)	(N/A)	(N/A)	
1 U1 Bot	0.00	(N/A)	0.00	(N/A)	M ₁ =	(N/A)	(N/A)	(N/A)	

9. Factored Loads and Moments with Corresponding Capacity Ratios

NOTE: Calculations are based on "Critical Capacity" Method.

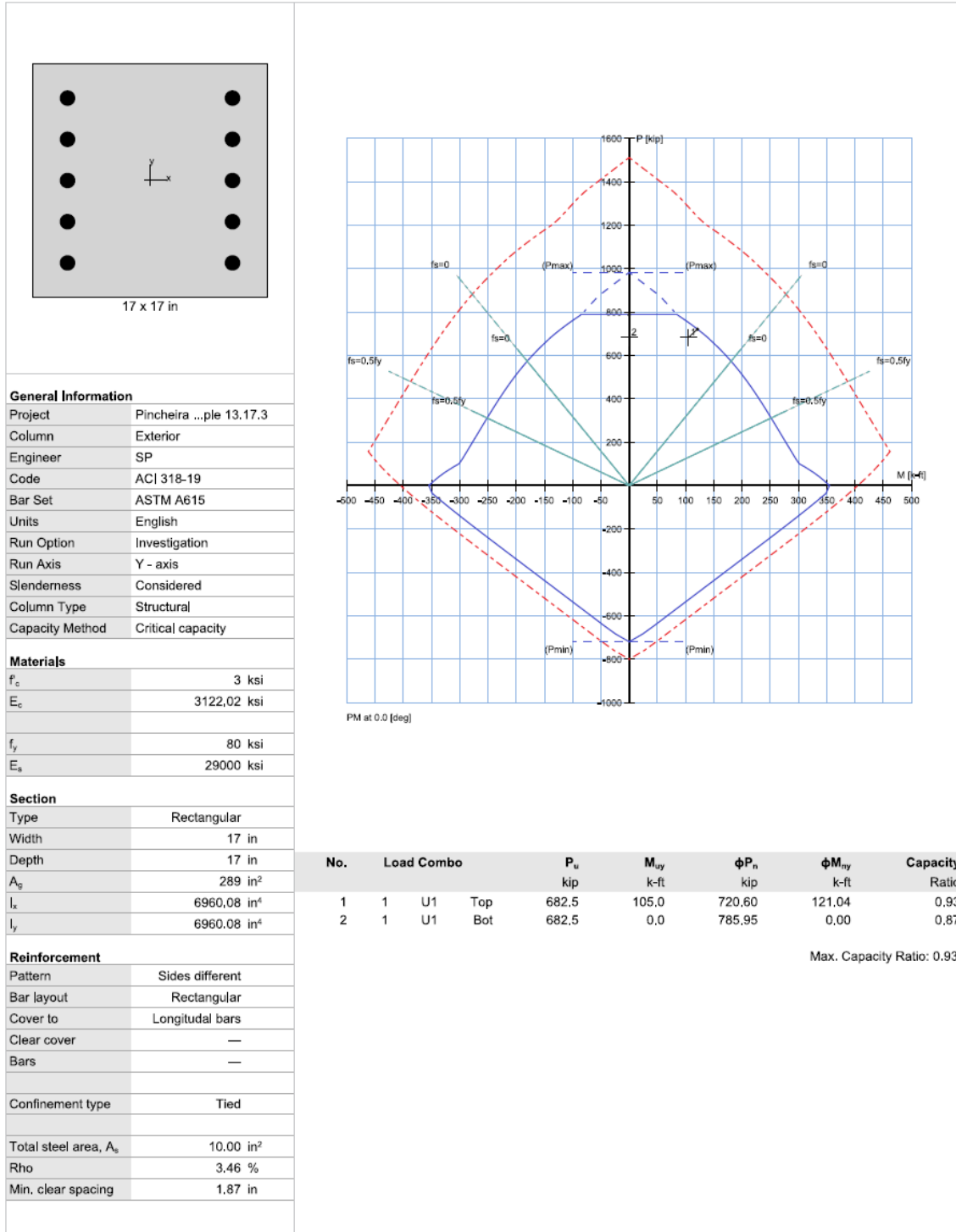
Each loading combination includes the following cases:

Top - At column top
Bot - At column bottom

No. Load Combo	Demand		Capacity		Parameters at Capacity			Capacity Ratio
	P _u kip	M _{uy} k-ft	ϕP_n kip	ϕM_{ny} k-ft	NA Depth in	ϵ_t	ϕ	
1 1 U1 Top	682.50	105.00	720.60	121.04	18.20	-0.00061	0.650	0.93
2 1 U1 Bot	682.50	0.00	785.95	0.00	19.95	-0.00082	0.650	0.87

10. Diagrams

10.1. PM at $\theta=0$ [deg]



6. Summary and Comparison of Design Results

Analysis and design results from the hand calculations above are compared with the exact values obtained from [spColumn](#) model.

Table 2 – Parameters for moment magnification of column in non-sway frame						
	k	(EI) _{eff} , kip.in. ²	P _c , kip	P _u , kip	δ _{ns}	M _c , kip.ft
Hand	0.959	10.56×10 ⁶	7871	525	0.66	105
spColumn	0.960	10.57×10 ⁶	7850	525	N/A	105

All the results of the hand calculations illustrated above are in precise agreement with the automated exact results obtained from the [spColumn](#) program. δ_{ns} in the methods of solution shown above need not be calculated since the slenderness effects need not be considered. The hand solution shows this value for illustration purposes.

7. Conclusions & Observations

7.1. General Observations

The analysis of the reinforced concrete section performed by [spColumn](#) conforms to the provisions of the Strength Design Method and Unified Design Provisions with all conditions of strength satisfying the applicable conditions of equilibrium and strain compatibility and includes slenderness effects using moment magnification method for sway and nonsway frames.

ACI 318 provides multiple options for calculating values of r and (EI)_{eff} leading to variability in the determination of the adequacy of a column section. Engineers must exercise judgment in selecting suitable options to match their design condition. The [spColumn](#) program utilizes the exact methods whenever possible and allows user to override the calculated values with direct input based on their engineering judgment wherever it is permissible.

7.2. Design Column Boundary Conditions in Slenderness Calculations

When the slenderness effects for a non-sway frame column is considered in creating a model using [spColumn](#), the effective length factor can be computed by defining the properties of the columns and beams connected to the top and bottom of the design column. The following notes are helpful when using [spColumn](#) to calculate the k value for some of the special boundary conditions cases:

- To model pin supports at the top and bottom of the design column:

Columns Above and Below

Column Above: No column specified
Height (c/c): 12 ft
Width (along X): 17 in
Depth (along Y): 17 in
Concrete, f'c: 3 ksi
Ec: 3122.1 ksi

Column Below: No column specified
Height (c/c): 12 ft
Width (along X): 17 in
Depth (along Y): 17 in
Concrete, f'c: 3 ksi
Ec: 3122.1 ksi

Y-Beams (perpendicular to Y)

Beam Location:
 Above Left Above Right
 Below Left Below Right

Beam Above Left:
 No beam specified Copy From Beam Right
Span (c/c): 30 ft f'c: 3 ksi
Width: 14 in Ec: 3122.02 ksi
Depth: 24 in Inertia: 16128 in⁴

Diagram: $\Psi_A = 999$, $\Psi_B = 999$, $k = 1.0$

“No column specified” for column above and column below

“No beam specified” for beams above and below

- To model fix supports at the top and bottom of the design column:

Columns Above and Below

Column Above: No column specified
Height (c/c): 12 ft
Width (along X): 17 in
Depth (along Y): 17 in
Concrete, f'c: 3 ksi
Ec: 3122.1 ksi

Column Below: No column specified
Height (c/c): 12 ft
Width (along X): 17 in
Depth (along Y): 17 in
Concrete, f'c: 3 ksi
Ec: 3122.1 ksi

Y-Beams (perpendicular to Y)

Beam Location:
 Above Left Above Right
 Below Left Below Right

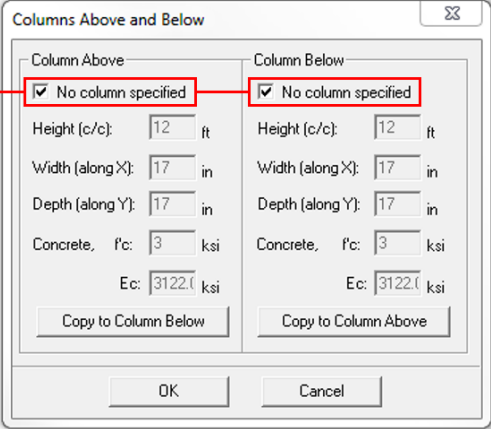
Beam Above Left:
 No beam specified Copy From Beam Right
Span (c/c): 30 ft f'c: 3 ksi
Width: 14 in Ec: 3122.02 ksi
Depth: 24 in Inertia: 1e+011 in⁴

Diagram: $\Psi_A = 0$, $\Psi_B = 0$, $k = 0.5$

“No column specified” for column above and column below

Define a high moment of inertia value for one of the beams at the top and bottom.

3. To model pin support at the top and fix support at the bottom of the design column:



Columns Above and Below

Column Above: No column specified

Column Below: No column specified

Height (c/c): 12 ft

Width (along X): 17 in

Depth (along Y): 17 in

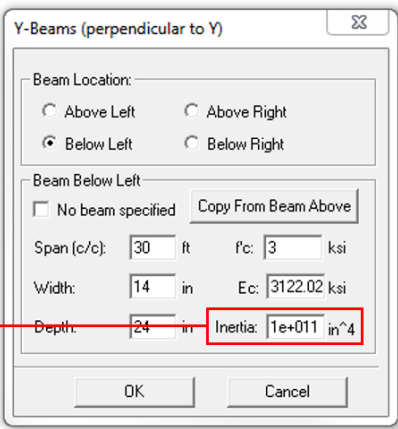
Concrete, f_c: 3 ksi

Ec: 3122.1 ksi

Copy to Column Below

Copy to Column Above

OK Cancel



Y-Beams (perpendicular to Y)

Beam Location:

Above Left Above Right

Below Left Below Right

Beam Below Left:

No beam specified

Span (c/c): 30 ft f_c: 3 ksi

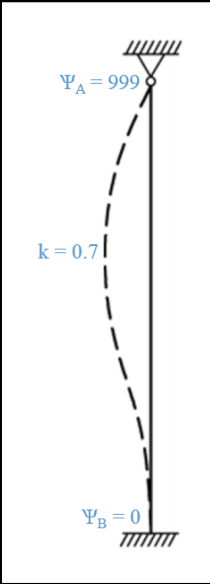
Width: 14 in Ec: 3122.02 ksi

Depth: 24 in **Inertia: 1e+011 in⁴**

OK Cancel

“No column specified” for column above and column below

Define a high moment of inertia value for one of the beams at the location of the fix support. “No beam specified” for beams at the location of the pin support.



$\Psi_A = 999$

$k = 0.7$

$\Psi_B = 0$